

Dynamics of a Particle

Kinematics

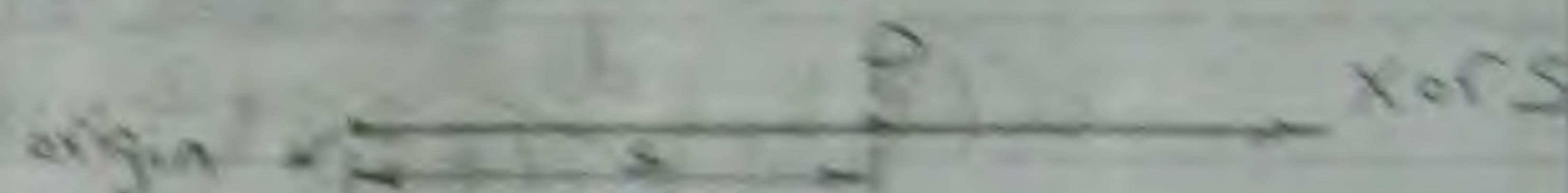
Kinetics

Kinematics of a particle:

- Properties of motion without taking reasons into consideration.

* Rectilinear motion

- In Figure:



Particle move along an horizontal axis

s : distance from a fixed point (origin)

s : represents position.

s is function of time as it varies with time $s(t)$

D : Displacement between time interval $(t_1 \text{ to } t_2)$



Displacement: $D = \Delta s = s_2 - s_1$

D : represents difference in position.

Velocity: The rate of change of position with time

$$v(t) = \frac{ds}{dt}$$

Acceleration: The rate of change of velocity with respect to time.

$$a(t) = \frac{dv}{dt}$$

s $\xrightarrow[\text{Integration}]{\text{Derivation}}$ v $\xrightarrow[\text{Integration}]{\text{Derivation}}$ a

* Jerk: change of acceleration with respect to time.

$$J(t) = \frac{da}{dt}$$

• It happens when acceleration changes
Random not smooth.

Example $s(t) = t^3 - 9t^2 + 15t$ m

Find 1) Initial velocity and acceleration.

2) The Time at which the velocity vanishes

$$v(t) = \frac{ds}{dt} = 3t^2 - 18t + 15 \quad (\text{m/s}) \quad (v=0)$$

$$a(t) = \frac{dv}{dt} = 6t - 18 \quad (\text{m/s}^2)$$

$$J(t) = \frac{da}{dt} = 6 \quad (\text{m/s}^3)$$

Answer 1) let $(T=0) \Rightarrow s_0 = 0$

$$v_0 = 15 \text{ m/s}, \quad a_0 = -18 \text{ m/s}^2$$

Notes

sign of acceleration: (-ve) \rightarrow to left, (+ve) to right

Type of acceleration ① accelerating rate
② decelerating rate.

\rightarrow accelerating is (+ve): when \vec{v} and \vec{a} in the same direction (The same sign)

\rightarrow decelerating is (-ve): when \vec{v} and \vec{a} in opposite directions (different signs).

\rightarrow momentarily stillness: The particle changes its direction of motion at that moment.

Answer ② : let $v=0 \Rightarrow 3t^2 - 18t + 15 = 0$ ($\div 3$)
 $t^2 - 6t + 5 = 0$

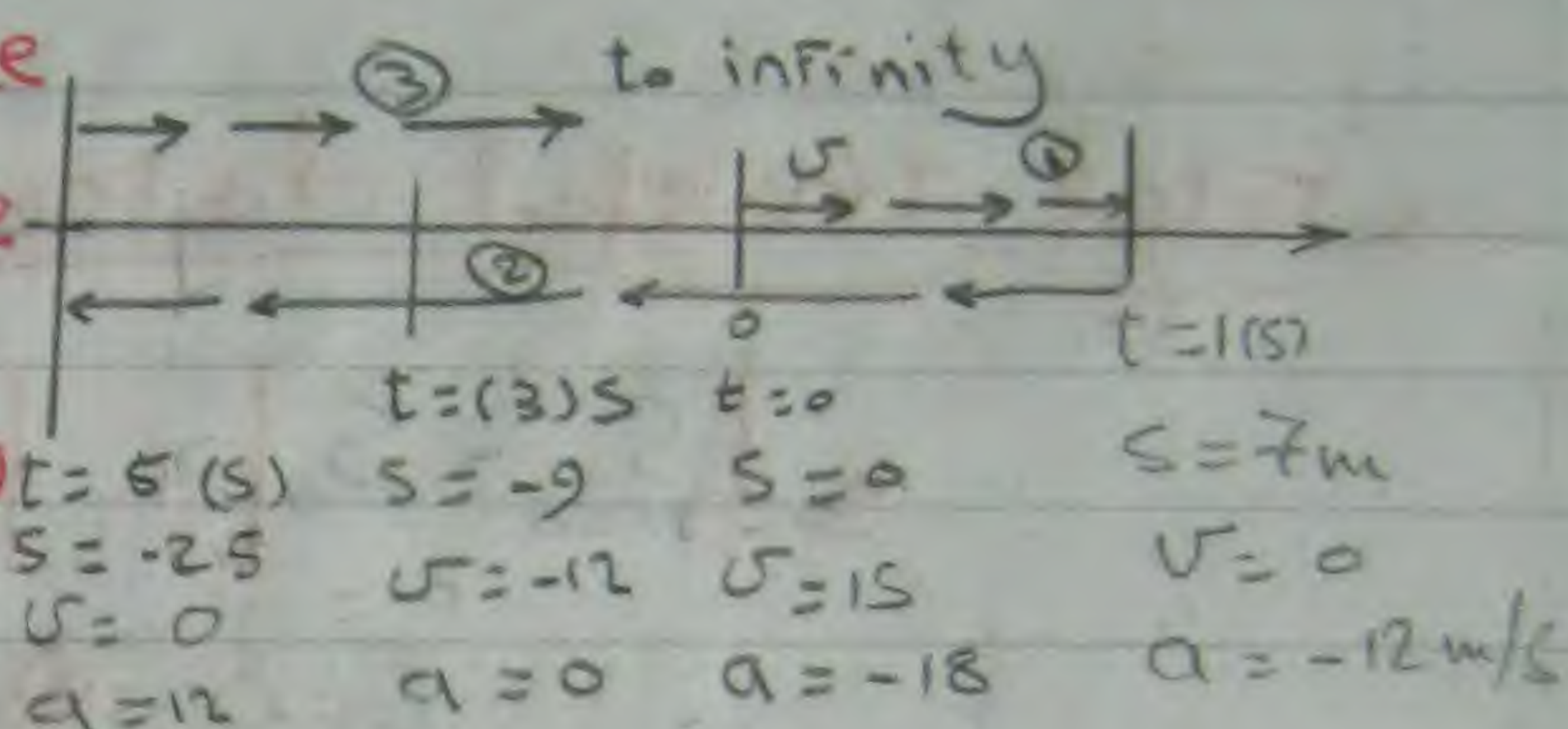
$(t-5)(t-1) = 0$

Hence, $t = 1(s)$, $t = 5(s)$

At $T=1(s)$ $S_1 = 7m$, $a_1 = -12m/s^2$.

At $T=5(s)$ $S_2 = -25m$, $a_2 = 12m/s^2$.

① The particle moves along the (+ve) direction to a distance 7m. ② moves to left to a distance of 25m along (-ve) axis. ③ moves to right.



3) The moment at which ($a=0$)

let $a=0 \Rightarrow 6t - 18 = 0 \Rightarrow t=3(s)$

$S|_{t=3} = -9m$, $v|_{t=3} = -12m/s$.

Note when $a=0$, It changes the type of velocity due to accelerating and decelerating.

when $v=0$, It changes the direction of motion.

4) at ($t=3$) Find the displacement and the total covered distance.

→ Displacement: $S_3 - S_0 = -9 - 0 = -9m$
 $t=0, t=3$ (From last to first position).

→ Total travelled distance: From ($0 \rightarrow 1$) = 7m
 $t=0 \rightarrow t=3$ From ($1 \rightarrow 3$) = 16m
 Total distance = $7 + 16 = 23m$.

(E) s) at $t = 6 \text{ (s)} \Rightarrow S_6 = -18 \text{ m}$.

→ Displacement: ($S_{\text{Final}} - S_{\text{Initial}}$)

$$\Delta = (1 - 1) (= S_6 - S_0) = -18 - 0 = -18$$

→ Total distance: $t(0 \rightarrow 1) = 7 \text{ m}$

$$t(1 \rightarrow 5) = 32 \text{ m}$$

$$t(5 \rightarrow 6) = 7 \text{ m}$$

$$\text{Total distance} = 7 + 7 + 32 = 46 \text{ m}.$$

→ Total traveled distance

$$= |S_{v=0} - S_0| + |S_{\text{Final}} - S_{v=0}|$$

OR

$$= |S_{v=0} - S_0| + |S_{v_2} - S_{v_1}| + |S_{\text{Final}} - S_{v=0}|$$

Note: Newton's law of acceleration used only in the case of constant acceleration.

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Section : 45

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